

Exercise 67

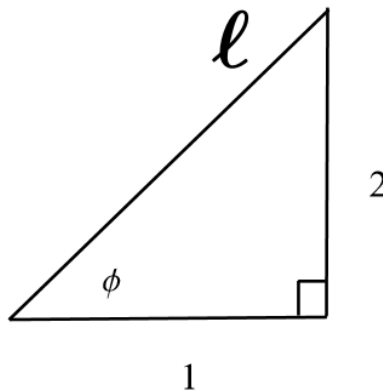
Let ℓ be the tangent line to the parabola $y = x^2$ at the point $(1, 1)$. The *angle of inclination* of ℓ is the angle ϕ that ℓ makes with the positive direction of the x -axis. Calculate ϕ correct to the nearest degree.

Solution

Determine the slope of the tangent line to the parabola at $x = 1$.

$$\begin{aligned} f'(1) &= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(1+h)^2 - 1^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{(1+2h+h^2) - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{2h+h^2}{h} \\ &= \lim_{h \rightarrow 0} (2+h) \\ &= 2 \\ &= \frac{2}{1} \end{aligned}$$

Slope is rise over run, so this determines two sides of a right triangle.



Tangent relates the angle with these two sides.

$$\tan \phi = \frac{2}{1} = 2 \quad \rightarrow \quad \phi = \tan^{-1}(2) \approx 63^\circ$$